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The weights of the rods are proportional to their lengths. $\therefore W:W'=l:m$.

$$z=\frac{l(PQ\cos\theta-\frac{1}{2}l\cos\theta)+m(PH\cos\phi-\frac{1}{2}m\cos\phi)}{l+m}$$

$$=[lr\sin\theta+lr\cos\theta\cot\tfrac{1}{2}(\phi+\theta)-\tfrac{1}{2}l^2\cos\theta+mr\sin\phi$$
$$+mr\cos\phi\cot\tfrac{1}{2}(\phi+\theta)-\tfrac{1}{2}m^2\cos\phi]/(m+l)\dots(1).$$

Also, $(\frac{1}{2}l\sin\theta+OI)l=(\frac{1}{2}m\sin\phi-OI)m$, or
$$2r(l+m)\sin\tfrac{1}{2}(\phi-\theta)=(m^2\sin\phi-l^2\sin\theta)\sin\tfrac{1}{2}(\phi+\theta)\dots(2).$$

Differentiating (1) and (2), we get

$$[l^2\sin\theta\sin^2\tfrac{1}{2}(\phi+\theta)-r(l+m)\cos\phi]d\theta$$
$$+[m^2\sin\phi\sin^2\tfrac{1}{2}(\phi+\theta)-r(l+m)\cos\theta]d\phi=0\dots(3).$$

$$\frac{2l^2\cos\theta\tan\frac{1}{2}(\phi+\theta)+l^2\sin\theta-m^2\sin\phi-2r(l+m)\cos\frac{1}{2}(\phi-\theta)}{\cos\frac{1}{2}(\phi+\theta)}d\theta$$

$$=\frac{2m^2\cos\phi\tan\frac{1}{2}(\phi+\theta)-l^2\sin\theta+m^2\sin\phi-2r(l+m)\cos\frac{1}{2}(\phi-\theta)}{\cos\frac{1}{2}(\phi+\theta)}d\phi\dots(4).$$

Eliminating $d\theta$, $d\phi$ between (3) and (4) we get, after reducing,

$$(m^2\sin\phi-l^2\sin\theta)^2\sin^2\tfrac{1}{2}(\phi+\theta)-4l^2m^2\sin^4\tfrac{1}{2}(\phi+\theta)$$
$$+r(l+m)(l^2\sin\theta-m^2\sin\phi)(\cos\theta-\cos\phi)+[2r(l+m)(l^2\cos^2\theta$$
$$+m^2\cos^2\phi)+r(l+m)(m^2\sin\phi+l^2\sin\theta)(\sin\phi+\sin\theta)]$$
$$\times\tan\tfrac{1}{2}(\phi+\theta)-4r^2(l+m)^2\cos^2\tfrac{1}{2}(\phi-\theta)=0\dots(5).$$

(2) and (5) determine the equilibrium. If $l=m$, $\phi=\theta$, and we get

$$l^2\sin^4\theta\cos\theta-2rl\sin\theta+4r^2\cos\theta=0.$$
$$(l\sin^3\theta-2r\cos\theta)(l\sin\theta\cos\theta-2r)=0.$$

$l\sin^3\theta=2r\cos\theta$ is the condition for equilibrium when the rods are equal.

214. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

An inelastic particle is projected in a direction BD from B in a straight line AB. It strikes a rigid line AD in D and returns to AB at C. Find AC/AB, and show on *a priori* ground that this ratio is independent of the velocity of projection.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let e be the elasticity of the rigid line AD, DR a prolongation of AD. Let $AB=a$, $\angle DAB=\beta$, $\angle DBC=\delta$, $\angle RDC=\theta$. Then $\angle ADB=\delta-\beta$, $\tan\theta=e\tan(\delta-\beta)$. $\therefore \theta=\tan^{-1}[e\tan(\delta-\beta)]$, and is known.

Since θ is independent of the velocity of projection, AC/AB is independent of this velocity.

$$AD : a=\sin\delta : \sin(\delta-\beta). \quad \therefore AD=\frac{a\sin\delta}{\sin(\delta-\beta)}.$$

$$AC : AD=\sin\theta : \sin(\theta-\beta). \quad \therefore AC=\frac{AD\sin\theta}{\sin(\theta-\beta)}=\frac{a\sin\delta\sin\theta}{\sin(\delta-\beta)\sin(\theta-\beta)}.$$

$$\therefore \frac{AC}{a}=\frac{AC}{AB}=\frac{\sin\delta\sin\theta}{\sin(\delta-\beta)\sin(\theta-\beta)}.$$

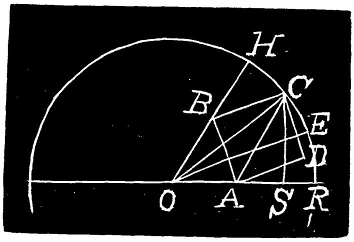
AVERAGE AND PROBABILITY.

193. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

What is the average area of all squares that may be inscribed in a given sector of a circle, a diagonal of the square being parallel to a random line across the sector?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

The following solution is for a sector less than a quadrant. Larger sectors would require a separate solution.



Let ROH be the given sector, $ABCD$ the inscribed square, C in the arc, A and B each in a radius; A in OR, B in OH; AC the diagonal representing the random direction of the line; CS, perpendicular from C on OR. All directions will be included between AC in OR and AC parallel to the bisector of sector.

Let $AB=x$, $OC=OR=r$, $\angle HOR=\beta$, $\angle CAR=\theta$. Then $AC=x\sqrt{2}$, $AS=x\sqrt{(2)}\cos\theta$, $CS=x\sqrt{(2)}\sin\theta$, $OA=x\sin(\frac{1}{4}\pi+\theta-\beta)/\sin\beta$.

$\therefore (x\sin(\frac{1}{4}\pi+\theta-\beta)/(\sin\beta+x\sqrt{(2)}\cos\theta)^2+2x^2\sin^2\theta=r^2$.

$$\therefore x^2=\frac{r^2\sin^2\beta}{\sin^2(\frac{1}{4}\pi+\theta-\beta)+2\sqrt{(2)}\sin\beta\cos\theta\sin(\frac{1}{4}\pi+\theta-\beta)+2\sin^2\beta}$$

$$=\frac{2r^2\sin^2\beta}{2+\sin2\beta-\cos2\beta+(1+\sin2\beta)\sin2\theta+(\cos2\beta-1)\cos2\theta}.$$